Statistics

Fall 2022

Lecture 28

Testing one Population mean μ:

S6.26

$H_0: \mu = \mu_0$	$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$
TTT	LTT	RTT

Case I: σ Known	Case II: σ Unknown
CV z invNorm	CV t invT $df = n-1$
CTS z → Z-Test	CTS t → T-Test
P-Value P inpt: Stats	P-Value P inpt: Stats

Use Testing chart for validity of H_0 or H_1.

Final Conclusion has to be about the claim

Reject the claim OR Fail-To-Reject the claim

Given $n=10$, $\bar{x}=82$, $s=7.5$, $H_0: \mu=85$

Claim is H_0, $\alpha=.1$

Test the claim.

$H_0: \mu=85$ claim

$H_1: \mu \neq 85$ TTT

Since σ is unknown

C.V. t invT $df=n-1=9$

TTT $\alpha=.1$

$t=$invT(.95, 9)=

CTS $t=-1.265$

P-value $P=.238$ VVV

T-Test

inpt:

$\mu_0: 85$

$\bar{x}: 82$

$s: 7.5$

$n: 10$

$\mu \neq \mu_0$ TTT

calculate

stats

CTS is in NCR H_0 valid

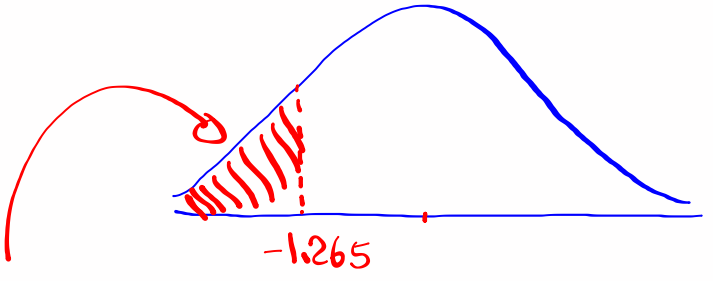
P-value $> \alpha$ $\Rightarrow$ H_1 invalid

valid claim

Fail-to-Reject the claim

CTS $t=-1.265$, $df=9$, TTT

find P-value.



P-value $= 2 * \text{Area}$

$= 2 * \text{tcdf}(-E99, -1.265, 9) = .238$

(L, U, df)

The College claims the mean Score on all final exams in a certain math class is at least 80. $\mu \geq 80$

I took a Sample of 20 final exams from that class, their mean Score was 78 with Standard deviation of 10. $n=20,\ \bar{x}=78,\ S=10$

Test the claim using $\alpha=.02$.

$H_0: \mu \geq 80$ claim

$H_1: \mu < 80$ LTT

σ Unknown

CTS $t=-.894$

P-value $P=.191$

T-Test

input: Stats

$\mu_0: 80$

$\bar{x}=78$

$S=10$

$n=20$

$\mu < \mu_0$ LTT

CV t invT LTT

$\alpha=.02$ $df=n-1=19$

H_1 CR .02

H_0 NCR .98

-2.205

$t=\text{invT}(.02, 19)=$

CTS is in NCR H_0 Valid

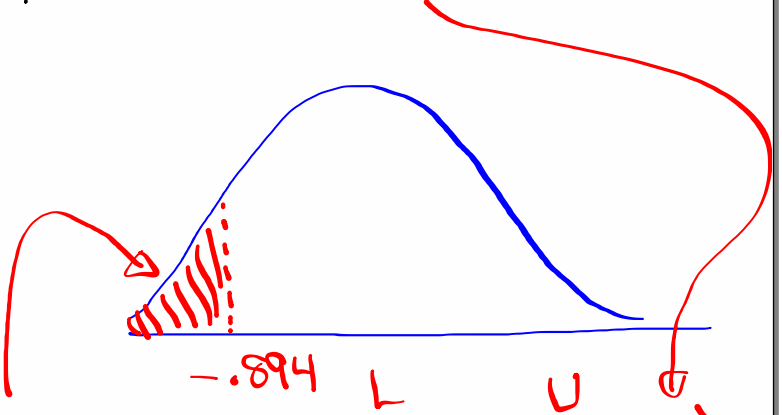
P-value $> \alpha$ $\Rightarrow$ H_1 Invalid

Valid claim

FTR the claim

CTS $t=-.894$ LTT $df=19$

Find P-Value.



P-value = Area = tcdf(-E99, -.894, 19)

= .191

I randomly Selected (15) students, and here are their ages: ↳ n=15

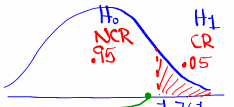
28	32	18	40	35
20	24	30	32	19
21	29	30	42	25

Find
1) $\bar{x} \approx 28.3$
2) $s \approx 7.3$
} Round to 1 decimal

The College claims that the mean age of all Students is at most 25 Yrs. $\mu \leq 25$

$H_0: \mu \leq 25$ claim
$H_1: \mu > 25$ RTT

Test the claim.
No $\alpha \rightarrow$ use .05
σ unknown
CV t invT RTT
$\alpha = .05$ $df = n-1 = 14$



$t = invT(.95, 14)$

CTS $t = 1.751$
P-value $P = .051$
T-Test

CTS is in NCR H_0 Valid
P-value $> \alpha \Rightarrow H_1$ invalid

Valid claim $\Rightarrow$ FTR the claim

inpt: STATS
$\mu_0 = 25$
$\bar{x} = 28.3$
$s = 7.3$
$n = 15$
$\mu > \mu_0$ RTT

If we change α to .06, .07, .08, .09, .1, ... then
P-value $\leq \alpha$
H_0 invalid $\&$ H_1 Valid
Invalid claim
Reject the claim

Testing One Population Standard deviation σ:

$H_0: \sigma = \sigma_0$	$H_0: \sigma \leq \sigma_0$	$H_0: \sigma \geq \sigma_0$
$H_1: \sigma \neq \sigma_0$	$H_1: \sigma > \sigma_0$	$H_1: \sigma < \sigma_0$
TTT	RTT	LTT

we use P-value method only.

CTS $\chi^2 = \dfrac{(n-1)\cdot s^2}{\sigma^2}$ then we use χ^2cdf to find the P-value.

$df = n-1$

we proceed with testing chart

Make final conclusion about the claim.

Given: $n=8$, $S=12$, $H_0: \sigma \le 10$ claim is H_0

$\alpha = .1$

$df = n-1 = 7$

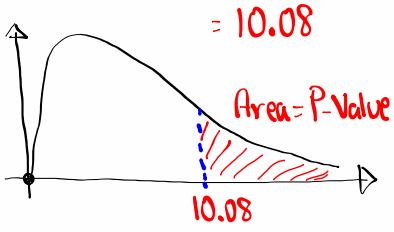
Test the claim.

$H_0: \sigma \le 10$ claim

$H_1: \sigma > 10$ RTT

CTS $\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(8-1)\cdot 12^2}{10^2}$

$= 10.08$



P-value $>$ α

.184 .1

H_0 valid, H_1 invalid

Valid claim

Fail-to-Reject the claim

P-value $= \chi^2cdf(10.08, E99, 7)$

$= .184$ ✓

The College claims that stand. dev. of ages of all students is at least 7.5 years.

$\sigma \ge 7.5$

I took a Sample of 10 students, and Standard deviation of their ages was 6.3 yrs.

$n=10$ $S=6.3$

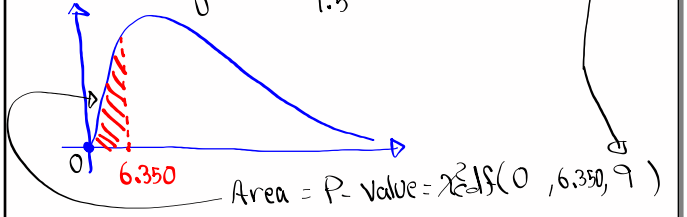
$df = n-1 = 9$

$H_0: \sigma \ge 7.5$ claim

Test the claim at $\alpha = .02$.

$H_1: \sigma < 7.5$ LTT

CTS $\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(10-1)\cdot 6.3^2}{7.5^2} = 6.350$



Area = P-Value $= \chi^2cdf(0, 6.350, 9)$

$= .296$

P-value $>$ α

.296 .02

H_0 valid ∴ H_1 invalid

Valid claim

FTR the claim

The college claims that standard deviation of all scores in all math classes is 10.

$H_0: \sigma = 10$ claim

$H_1: \sigma \neq 10$ TTT

I took a sample of 12 math exams, and standard deviation of their scores was 6.

$n = 12$ $S = 6$

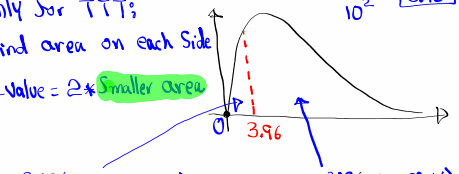
Test the claim. $df = n-1 = 11$ CTS $\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2}$

No $\alpha \Rightarrow \alpha = .05$ $= \frac{(12-1)\cdot 6^2}{10^2} = 3.96$

Only for TTT;

Find area on each side

P-value = 2 * Smaller area



$\chi^2 cdf(0, 3.96, 11)$ $\chi^2 cdf(3.96, E99, 11)$

$= .029$ $= .971$

P-value $= 2(.029) = .058$

P-value $> \alpha$ H_0 valid $\Rightarrow$ valid claim $\Rightarrow$ FTR the claim

$.058 > .05$ H_1 invalid

If we change α to .06, .07, .08, .09, .1, ...

then P-value $\leq \alpha$ H_0 invalid, H_1 valid $\Rightarrow$ invalid claim $\Rightarrow$ Reject the claim

Given: $n = 6$, $S = 9$, $H_1: \sigma \neq 5$, $\alpha = .1$ $df = n-1 = 5$

H_0 is the claim

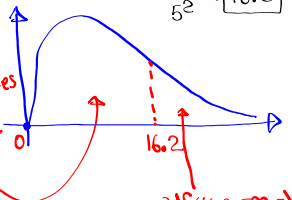
Test the claim. CTS $\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2}$

$H_0: \sigma = 5$ claim $= \frac{(6-1)\cdot 9^2}{5^2} = 16.2$

$H_1: \sigma \neq 5$ TTT

Find area on both sides

P-value = 2 * Smaller area



$\chi^2 cdf(0, 16.2, 5) = .994$ $\chi^2 cdf(16.2, E99, 5)$

$= .006$

P-value $= 2 * (.006) = .012$

P-value $\leq \alpha$ H_0 invalid $\Rightarrow$ invalid claim

$.012 < .1$ H_1 valid Reject the claim

If we change α to .01, then

P-value $> \alpha$ H_0 valid $\Rightarrow$ valid claim

$.012 > .01$ H_1 invalid Fail-to-Reject the claim